J. Engg. and Appl. Sci. Vol. 34 No. 2 July - December 2015 *ISSN 1023-862X*

SINGLE FACILITY LOCATION SELECTIONPROBLEM FOR A PAKISTAN BASED ICE CREAM COMPANY: A CASE STUDY

Suhaib*, Shahid Maqsood*, Sikandar Bilal Khattak*, Misbah Ullah*, Rehman Akhtar*, Rashid Nawaz*, Iftikhar Hussain*, Abdul Shakoor**, Khizar Azam**

ABSTRACT

The facility location optimization problems are economically justifiable and play a critical role in reducing the overall running cost. An optimal facility location is directly proportional to traveling cost between facility and destinations. In past decade, extensive research has beencarried out in this field. Most of the facility location optimization techniques provide optimum or near optimumsolution. In this paper, an optimization technique median method is applied to a real world problem of a multinational ice cream company's factory located in Lahore, Punjab (province) Pakistan, with an objective of minimum overall cost between a warehouse and destinations. The company wanted to locate the warehouse in Khyber Pakhtunkhwa (KPK), province in order to meet the demand in KPK on time and reduce the travelling cost between the warehouse and destinations. Moreover, using Contour line technique a generic computer based simulation model is developed using MATLAB in order to draw an ISO cost contour line around the optimum location. The model is successfully developed and provides encouraging results. The cost analysis of each potential location is also presented in this paper.

KEYWORDS: *Facility location, optimization, p-median, Euclidian, contour lines, feasible locationMaintenance Prevention, Heavy Earth Moving Equipment, Checklist.*

INTRODUCTION

Facilities can be broadly classified as a critical component of strategic planning where 3Ms (Man, Material, and Machines) which are integrated for the purpose of making a tangible product or provide a service[1]. A facility if properly managedhave a tendency to achieve its stated purpose i-e to satisfymultiple objectives such as the production of a product or providing a service of high quality at lower cost using the least amount of resources[2]. Facility location selection problem is at the heart of supply chain management. The reduction of travelling distances is the key to almost all supply chains and is critical in many cases such as health care and rescue facilities. A survey conducted by Zimmerman and Lebeau[3] discovers that a patient's decision to have prostate cancer screening is affected by theconvenience to the medical facility. The reduction in distances is directly affecting the overall cost[4]. The facility location analysis is aimed to achieve cost effective installation of facilities taking into consideration the location of markets (customers) and raw materials (suppliers). The accessibility to skill and info have added important strategic aspects affecting location decisions[5-7].

LITERATURE REVIEW

In literature, various researcher[4,8-17], have presented various decision making models in this area of research.

Farahani, et al.[18] carried out an extensive review of the Facility Location Planning (FLP) models and their solution and concluded that among the FLPs models, covering problem is the most popular model. This traditional model is popular due to its practical applications in real-world life such as distribution management, transportation, and health and telecommunication networks design. Covering problem is a computational minimization problem, which decides that whether a combinatorial structure covers another or how large the structure has to be to do that?[19]

Facility location problem is broadly classified into single or multiple facility location problem or continuous and discrete location problems Tompkins, White et al[20]. Discrete location problems are further classified p-centre, p-median, simple plant, and quadratic assignment location problem sets[21].

In location theory the minisum location-allocation problem is one of the larger family of problems. Rather than finding the centre among points as in the p-centre

* *Department of Industrial Engineering, University of Engineering and Technology, Peshawar*
** *Department of Mechanical Engineering, University of Engineering and Technology, Peshawar*

problem (where the objective is to decrease the maximum distance) the p-median problem find medians among points. The minisum problem can be traced back to 17th century with Fermat's question: in a given plane, locate a median such that the sum of distances from each corner of the triangle i-e the 3 points are minimum from the median.

The work of Alfred Weber[22] was the earliest contribution in the field of location analysis problems. He measured the location between two resources and single market on a map. The Weber problem is the 1-median problem on a plane (continuous feasible space) which is usually accredited as the first location-allocation problem in location theory. Later the Weber problem was generalized and extended to problems with locating the median for more than three points in the plane points and for multi-facility location problem. The multi-facility problem was generalized later for the case of $p > 1$ medians among a number of points in the plane[23]. The Weber problem uses Euclidean plane to locate medians (facilities)[24]and[25] used generalised Weber's problem (or Fermat or Steiner problem) and solved the problems using independently considered iterative process technique. Using rectilinear distance, the multi-facility location problem was solved using a network flow procedure by[24,26,27,28].

The p-median problem was first developed by Hakimi[29,30] first developed the p-median problem, for finding the medians on a network/graph. J Reese's review Reese[23], shows the similarity between Hakimi's absolute median and Weber's weighted problem. Reese states that Hakimi defines his median problem as: in a given network or graph, consider a point at which the sum of distances between that particular point and the vertices of the graph are minimum. It was proved that there will always be a median on the vertex which will be optimum, irrespective of the location of the point at the graph. Hence represents a continuous problem in a discrete manner.

Hakimi[30] presented a general model for the absolute median problem which seeks to minimize the sum of weighted distances in order to find p medians on a graph. The solutions at ***p*** vertices were called p-medians. Unlike the weber problem Hakimi's p median problem is defined on a graph or a network, rather than a plane.

In order to find the optimal location of switching centre in a communication network the absolute median and p-medianproblem were used.

Since the problem at hand is a continuous problem (p-median), that has been made discrete because of finite number of locations to choose from. The methods applied to solve it are (a) Euclidean distance location method (b) Squared Euclidean distance location method and (c) Rectilinear distance location method. In this paper the rectilinear distance is used for finding an optimal location along with median method.

Hermann Minkowski, a German, in nineteenth century considers Taxicab geometry which is also known as rectilinear distance, city block distance, Manhattan distance, or the length of Manhattan. The name comes from the grid layout of the streets of Manhattan Island. In rectilinear distance the sum of the absolute differences of their Cartesian coordinates is equal to the distance between two points. It is used for finding optimum location along rectilinear paths.

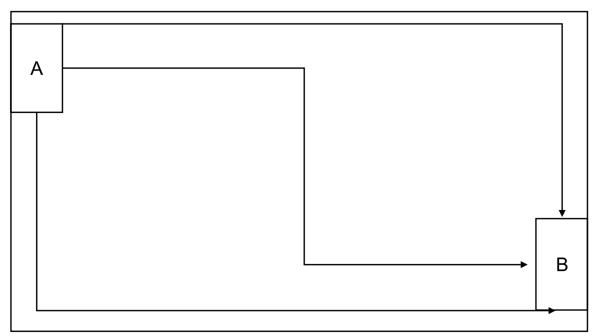

Figure 1. Representation of rectilinear paths

Consider the points A and B in Figure 1, there are different paths between A and B, for which the rectilinear distance, is the same. Rectilinear distance possess infinite number of paths as oppose to Euclidean distance where there is only a unique path.

Mathematically

$$\min_{x,y} F(x, y) = \sum_{i=1}^{m} wi\,|x - \alpha_i| + |y - b_i| \qquad (1)$$

From Equation (1) it is seen that the problem can also be written as

$$\min_{x,y} F(x,y) = \min_x \sum_{i=1}^{m} wi\,|x - \alpha_i| + \min_y \sum_{i=1}^{m} wi\,|y - b_i| \quad (2)$$

The right side of Equation (2) can be treated separately as an optimization problem.

$$\min_x f1\,(x) = \sum_{i=1}^{m} w1\,|x - \alpha_i| \quad (3)$$

$$\min_y f2\,(y) = \sum_{i=1}^{m} w1\,|y - b_i| \quad (4)$$

The problem of locating the rectilinear distance is a Multi Facilities Location (MFL) variant where p = 1. The rectilinear distance method is considered best in cases where the distance is along rectangular paths such as a grid of city streets or along a network of corridors expanded through a factory or a warehouse[31]. The rectilinear location Problem for single facility was first solved by Francis in 1963, in which a simple median problem was considered for optimal location. Later on for the multi facility rectilinear distance location problem a special case of was solved Francis[32], which dealt with equal weights.

In this paper, a model for real world location selection problem is proposed. The proposed model can handle a large number of alternatives locations and destinations.

Design and modelling of location selection problem

A multinational ice cream company wants to locate a warehouse in KPK, province in order to fulfill the ice cream demand for the entire province in time and in a cost effective way. Currently, the company is paying a huge cost on logistics to meet the customers demand in terms of transportation cost from its production plant in Punjab tothe warehouses in KPK. The company is planning to place the warehouse in such a location from where it can meet the demand from KPK with minimum transportation cost and hence overall cost.

For the formulation of this location selection problem for ice cream Company, an actual data from the company for activities and specified location is used Table 1, shows the actual data in terms of demand for each city and number of locations. The distances shown from factory to destinations are taken from maps obtained from government department of highways. The figure also shows that the highest demand is for Peshawar which is 277200 liters. While the lowest demand is for Thal city Table 1 also shows the distances between the Lahore (Factory) and destinations. The longest and shortest distance is between Lahore to Bhakkar and Lahore to Timurgara, respectively.

The primary objective of this paper is to find an optimal or near optimal warehouse location for and if the optimal location in not available or otherwise occupied then to draw an ISO cost contour line around the optimal location.

Table 1. Total distance from Lahore to destinations and demand for each destination

S. No	City	Total distance from Lahore to destinations(km)	Total Demand(liters)
1	Bannu	455	12000
2	Bhakkar	422	7000
3	Bhatkhela	532	4000
4	Charsadda	490	20000
5	D.I Khan	429	13000
6	Hangu	484	6000
7	Karak	473	8000
8	Kohat	443	10500
9	Mardan	473	20000
10	Nowshera	476	20000
11	Peshawar	513	277200
12	Swabi	434	20000
13	Swat	578	20000
14	Thal	540	3500
15	Timurgara	579	9000

Development steps of location Model

Mathematically the problem can be represented as

$$\text{Minimize} \sum_i \sum_j w_i\, d_i\, Y_{ij} \quad (5)$$

$$\text{Subject to} \sum_j X_j = P \quad (6)$$

$$\textstyle\sum_j Y_{ij} = 1 \quad \forall i \quad (7)$$

$$Y_{ij} - X_j \leq 0 \quad \forall i, j \tag{8}$$

$$X_j \in \{0,1\} \ \forall i \tag{9}$$

$$Y_{ij} \in \{0,1\} \ \forall i,j \tag{10}$$

Where

i = Demand node

j = Potential facility site

wi = Demand at node

dij = Distance between and

P #of facilities that are to be located

Xj = {1 if we tocate at potential facility site j 0 otherwise}

Yij = {1 if demands at node i are served by a facility site j 0 otherwise}

The objective function (5) is used to minimize the total travelling distance while the demand is placed as the weight on axis. The number of facilities p required is shown by constraint (6). Weights are assigned to facilities using equation (7). Constraint (8) allows the demand to be assigned to only some existing facility. (9) And (10) are binary requirements for the problem variables.

Assuming that only one facility is to be located i-e p=1 and the demand from that facility to all the customer locations are fulfilled and the facility is available (open) at any time.

METHODOLOGY

Median method is used to find the optimal location. The procedure for median method is given below.

Median method

The existing facilities are arranged in an increasing order of their x coordinates. Half of the cumulative weightsaretaken; thej^{th} element of x coordinate must be equalto or should exceed half of the cumulative weight.

$$\sum_{i=1}^{J-1} w_i < \sum_{i=1}^{m} \frac{w_i}{2} \ and \ \sum_{i=1}^{J} w_i \geq \sum_{i=1}^{m} \frac{w_i}{2} \tag{11}$$

The procedure is then repeated to find the y coordinates.

$$\sum_{i=1}^{k-1} w_i < \sum_{i=1}^{m} \frac{w_i}{2} \ and \ \sum_{i=1}^{k} w_i \geq \sum_{i=1}^{m} \frac{w_i}{2} \tag{12}$$

The optimal facility coordinates should have the same x and y coordinate as that of some existing facility. Contour Lines.

An ISO cost contour is a closed path around the optimum point along which the cost function remains constant. The location on the contour is not the optimum but it is near optimum and often times the most feasible location.

Procedure

Steps for constructing contour lines for the median method are given below:

1. Plot points and draw lines through these points perpendicularly. The lines should be parallel to the x axes and y axes

2. Moving from left to right name the lines parallel to y axes as.

3. Similarly moving from top to bottom, the lines parallel to x axis are numbered as.

4. The of the is denoted as.

5. And of is denoted as.

6. Denote the region enclosed by vertical lines and horizontal lines. (for all the regions to be numbered imagine there is a vertical line numbered 0 at the left of vertical line 1, now a vertical line which is numbered must be at the right hand side of vertical line, similarly a horizontal line which is and is, and a horizontal line which is denoted by).

7. The is assigned with weights.

8. Now for determining the values.

$$M_0 = -\sum_{j=1}^{p} C_j = \sum_{i=1}^{m} w_i \quad (13)$$

$$M_1 = M_0 + 2C_1 \quad (14)$$

$$M_2 = M_1 + 2C_2 \quad (15)$$

$$M_p = M_p - 1 + 2C_p = \sum_{i=1}^{m} w_i \quad (16)$$

$$N_0 = -\sum_{j=1}^{q} D_i = \sum_{i=1}^{m} w_i \quad (17)$$

$$N_1 = N_0 + 2D_1 \quad (18)$$

$$N_2 = N_1 + 2D_2 \quad (19)$$

$$N_q = N_q - 1 + 2D_q = \sum_{i=1}^{m} w_i \quad (20)$$

And place ***M*** values along x-axis and ***N*** vales along y-axis

9. The slope S*ij* of any contour line passing through region *[i,j]* is computed as follows:

$$S_{ij} = -\frac{M_i}{N_i} \quad (21)$$

RESULTS AND DISCUSSIONS

Table 2 shows the overall results in tabulated form. Column 1, 2 and 3 shows the number of cities, name of cities and total distance between destination and factory based in Lahore in km. Column 5 shows the total demand for each city in litres. The demand for Peshawar is higher than the rest of cities i.e. 277200 litres. Column 4 presents the total cost from a potential optimum location candidate and other candidates as destinations. This cost is calculated using equation (22). For example, if city number 3, Swabi is considered as optimum location the total cost between Swabi and all destination cities is 50490 Rupees. This distance is calculated using the median method. Table 2 listed all the distances calculated using same technique. It shows that Peshawar is with the minimum cost followed by Charsadda and Kohat.

Cost = Distance x Average fuel price x 2 (two way) (22)

Figure 2 illustrates Peshawar as the optimal location found. This is because Peshawar has the highest demand.

Table 2. Total cost when each destination is assumed optimum

S.no	City	Total distance from origin Lahore (km)	Total cost when it is considered optimum (Rupees)*	Total demand (Liters)
1.	Peshawar	513	36758	277200
2.	Nowshehra	476	39508	20000
3.	Swabi	434	50490	20000
4.	Kohat	443	38610	10500
5.	Swat	578	59913	20000
6.	Mardan	473	39398	20000
7.	Bhatkhela	532	48913	4000
8.	Timurgara	579	66513	9000
9.	Charsadda	490	37565	20000
10.	Karak	473	45247	8000
11.	Hangu	484	43798	6000
12.	Bhakkar	422	87285	7000
13.	D.I Khan	429	80007	13000
14.	Bannu	455	53222	12000
15.	Thal	540	51883	3500

*Total cost is rounded off to two decimal places

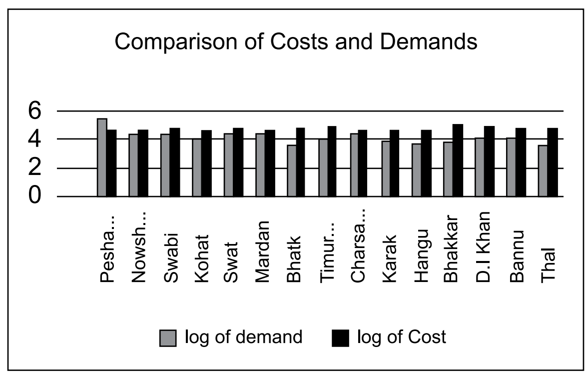

Figure 2. Bar graph showing that Peshawar is the optimum location for placing the warehouse

The figure is represented with log values as Peshawar has a very demand compared to other cities. The cost will increase as the optimal location is changed from Peshawar. A contour line is drawn around the optimum location in case of the unavailability of the optimal location. The contour line is ISO cost. Hence every location on the contour line has the same cost

Using MATLAB, a tool is developed that gives results

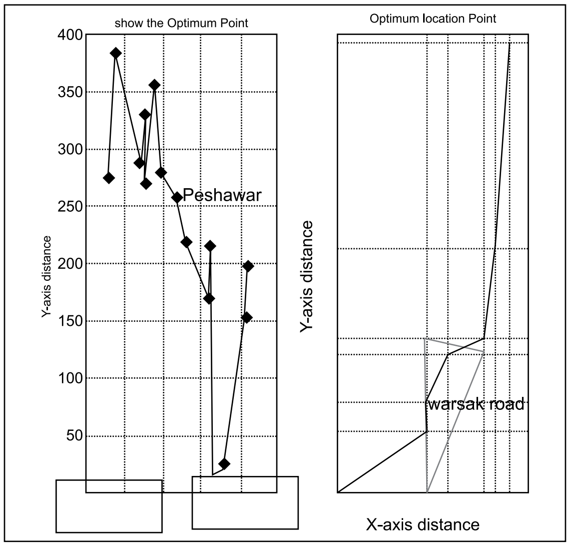

Figure 3. A graphical illustration of the results by the tool

using median problem algorithm. The tool also draws a contour around the optimal location which is ISO cost at any point on the contour. A screen shot of graphical representation tool is shown in Figure 3. This tool uses median algorithm and computes the optimum location by the travelling distances as well as by taking into consideration the demand and their weights assigned. Since, Peshawar has the maximum demand and is optimum location discussed earlier. Peshawar is a big city the tool pin points the location for warehouse within Peshawar city onwarsakroad as shown in the Figure 3. Although, this is a feasible location with available sites for warehouses the tool still provides the user anISO cost contour line around the optimum. This enables the user to choose the near-optimal or feasible location near optimum location in case the optimum is not feasible.

CONCLUSIONS

The basic objective of this study was to locate a warehouse such that the total travelling distance between the warehouse and all of the destination locations (represented by demand nodes n and weights w) is minimized. The research objective for this p-median problem was successfully achieved. The secondary objective achieved was development of a generic tool for single facility location. The tool minimizes the travelling distance between any number of locations and destinations and provides the user with an optimum location. In case the optimum location is infeasible such as a location on a river or the location is otherwise occupied the tool draws a contour line around optimum location which serves as an alternate to the optimum location. In future, this tool with a little modification can be applied to multi-facility location problem with infinite number of destinations.

REFERENCES

1. *Owen, S.H. and Daskin, M.S. 1998. Strategic facility location: A review. European Journal of Operational Research, 111(3): p. 423-447.*

2. *Tompkins, J.A., White, J.A., Bozer, Y.A., and Tanchoco, J.M.A. 2010, Facilities Planning. 4 ed. Wiley.*

3. *Zimmerman, B.J. and R.B. Lebeau 2000, A commentary on self-directed learning. Problem-based learning: A research perspective on learning interactions, p. 299-313.*

4. *Dantrakul, S., C. Likasiri, and R. Pongvuthithum 2014, Applied p-median and p-center algorithms for facility location problems. Expert Systems with Applications, 41(8): p. 3596-3604.*

5. *Ferdows, K. 1997, Made in the world: the global spread of production. Production and Operations Management, 6(2): p. 102-109.*

6. *Vereecke, A. and S. Muylle 2006, Performance improvement through supply chain collaboration in Europe. International journal of operations & production management, 26(11): p. 1176-1198.*

7. *Feldmann, A. and J. Olhager 2009, Plant roles and decision-making in manufacturing networks. in Proceedings 16th annual EurOMA Conference, Gothenburg.*

8. *Abravaya, S. and M. Segal 2009, Low complexity algorithms for optimal consumer push-pull partial covering in the plane. European Journal of Operational Research, 197(2): p. 456-464.*

9. *Avella, P., M. Boccia, S.Salerno, and I. Vasiltev (2012) An aggregation heuristic for large scale<i>*

p</i>-median problem. Computers & Operations Research, 39(7): p. 1625-1632.

10. *Berman, O., Z. Drezner, and D. Krass 2010, Generalized coverage: New developments in covering location models. Computers & Operations Research, 37(10): p. 1675-1687.*

11. *Canbolat, M.S. and G.O. Wesolowsky 2010, The rectilinear distance Weber problem in the presence of a probabilistic line barrier. European Journal of Operational Research, 202(1): p. 114-121.*

12. *Current, J.R., C.S. Re Velle, and J.L. Cohon 1985, The maximum covering/shortest path problem: A multiobjective network design and routing formulation. European Journal of Operational Research, 21(2): p. 189-199.*

13. *Hsieh, K.-H. and F.-C. Tien 2004, Self-organizing feature maps for solving location–allocation problems with rectilinear distances. Computers & Operations Research, 31(7): p. 1017-1031.*

14. *Kalcsics, J., S. Nickel, M.A. Pozo, J. Puerto, and A.M. Rodriguez-Chia (2011) The multicriteria p-facility median location problem on networks. European Journal of Operational Research, 235(3) p. 484-93.*

15. *Sevkli, M., R. Mamedsaidov, and F. Camci 2014, A novel discrete particle swarm optimization for p-median problem. Journal of King Saud University - Engineering Sciences, 26(1): p. 11-19.*

16. *Yaghini, M., M. Karimi, and M. Rahbar 2013, A hybrid metaheuristic approach for the capacitated p-median problem. Applied Soft Computing, 13(9): p. 3922-3930.*

17. *Sherali, H.D., S. Ramachandran, and S.-i. Kim 1994, A localization and reformulation discrete programming approach for the rectilinear distance location-allocation problem. Discrete Applied Mathematics, 49(1): p. 357-378.*

18. *Farahani, R.Z., N. Asgari, N. Heidari, M. Hosseininia and M. Goh (2012) Covering problems in facility location: A review. Computers & Industrial Engineering, 2012. 62(1): p. 368-407.*

19. *Vazirani, V.V. 2001 Approximation algorithms. Springer-Verlag New York, Inc. 378.*

20. *Tompkins, J., J. White, Y. Bozer, and J. Tanchoco (2010) Facilities planning 4th edition. Hoboken: John Wiley & Sons.*

21. *Krarup, J. and P.M. Pruzan, Jonathan Halpern 1983 European Journal of Operational Research, 12(1): p. 1-2.*

22. *Weber, A. 1929 Theory of the Location of Industries [translated by CJ Friedrich from Weber's 1909 book]. Chicago: The University of Chicago Press.*

23. *Reese, J., 2006Solution methods for the p-median problem: An annotated bibliography. Networks, 48(3): p. 125-142.*

24. *Cooper, L., 1967 Solutions of generalized locational equilibrium models†. Journal of Regional Science, 7(1): p. 1-18.*

25. *Kulin, H.W. and R.E. Kuenne 1962 An efficient algorithm for the numerical solution of the generalized Weber problem in spatial economics. Journal of Regional Science, 4(2): p. 21-33.*

26. *Cooper, L., 1972 The transportation-location problem. Operations Research, 20(1): p. 94-108.*

27. *Katz, I.N. and L. Cooper 1981 Facility location in the presence of forbidden regions, I: Formulation and the case of Euclidean distance with one forbidden circle. European Journal of Operational Research, 6(2): p. 166-173.*

28. *Cabot, A.V., R.L. Francis, and M.A. Stary 1970 A network flow solution to a rectilinear distance facility location problem. AIIE Transactions, 2(2): p. 132-141.*

29. *Hakimi, S.L. and S.S. Yau, 1964. Distance matrix of a graph and its realizability. Quart. Appl. Math, 22: p. 305-317.*

30. *Hakimi, S.,1965 Optimum distribution of switching centers in a communication network and some related graph theoretic problems. Operations Research, 13(3): p. 462-475.*

31. *Francis, R.L., 1963 A note on the optimum location of new machines in existing plant layouts. J. Indust. Engrg, 14: p. 57-59.*

32. *Francis, R.L., 1964. On the location of multiple new facilities with respect to existing facilities. J. of Industrial Engineering, 15(196): p. 106-107.*